

Monte Carlo Simulation

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Introduction

Comprehending and communicating various types of risk is one of the most challenging tasks facing advisors before, during, and after the planning process. With the investors' level of sophistication increasing, advisors confront difficult issues each day in understanding and conveying risk effectively.

Mean-Variance Optimization in EnCorr Optimizer

Ibbotson Associates creates an efficient frontier using a technique known as mean-variance optimization (MVO). The efficient frontier is derived from the underlying asset classes' expected return, standard deviation, and correlations (Figure 1). MVO is the process of identifying portfolios that have the highest possible return for a given level of risk, or the lowest possible risk for a given return.

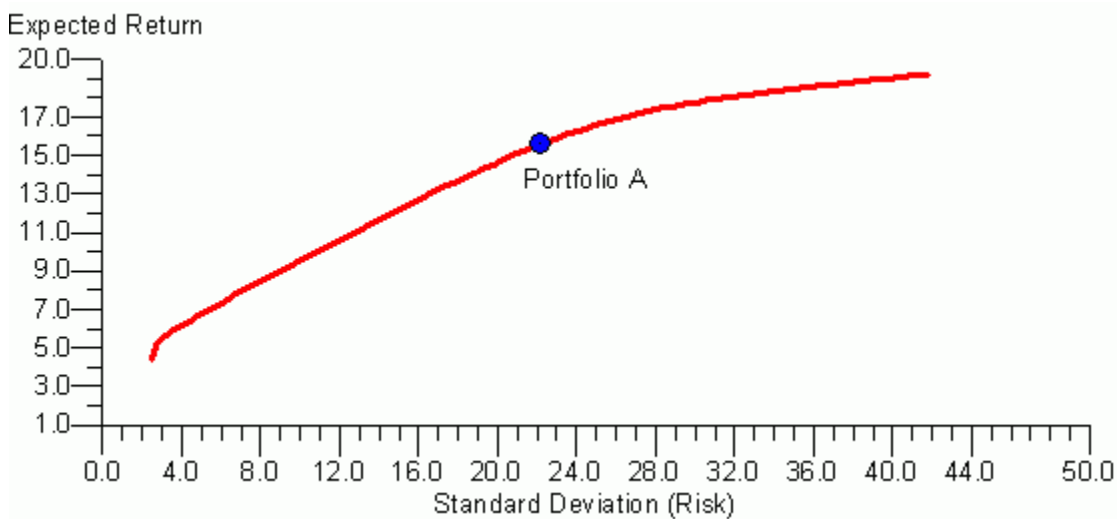


Figure 1 Efficient Frontier

Once you analyze the efficient frontier, you may want to look at specific asset mixes (i.e., Portfolio A shown in Figure 1) and their characteristics. In EnCorr Optimizer it is possible to view a variety of statistics including wealth and return percentiles tables (Figures 2 and 3). These tables allow the user to see the probability of achieving certain wealth and return values for a variety of time horizons. The problem is that MVO is predicated on the proposition that the uncertainty facing the investor is known with precision. The expected values, standard deviations, and correlations of those expectations are assumed to be known exactly. In reality, these statistical parameters that describe the uncertainty involved are only approximately known. It is necessary to have a tool like Monte Carlo simulation that factors in the uncertainty the future holds.

What is Monte Carlo Simulation?

Monte Carlo simulation is a problem-solving technique utilized to approximate the probability of certain outcomes by performing multiple trial runs, called simulations, using random variables. The probability distribution of the results is calculated and analyzed in order to infer which values are most likely to be produced. It's similar to variables that have a known range of values but an uncertain value for any particular time or event (e.g., interest rates, stock prices, weather conditions, insurance, etc.).

Monte Carlo Simulation and Asset Allocation

Real-life investing decisions involve all sorts of aspects, ranging from saving, to spending, to inflation, and more. When all of these complexities need to be considered, Monte Carlo simulation can be quite useful. The process starts with a set of assumptions about the estimated mean, standard deviation, and correlations for a set of asset classes or investments. These assumptions are used to randomly generate thousands of possible future return scenarios—somewhat similar to drawing numbers out of a hat. These returns can then be used in conjunction with a client's year-by-year cash flows and asset allocation. Monte Carlo techniques can calculate and display risk in a personalized way that is easy for investors to understand. The results from simulation are often used to construct and evaluate the appropriate asset allocation policy.

Ibbotson's Method for Simulation Is Parametric

Parametric simulation is based on a user providing a mean, standard deviation, and correlations for the assets being used. Once these parameters are set, a computer program is used to generate random samples from the distribution these parameters define.

This provides a much richer set of results since the program can draw any number under the curve, not just the numbers that have occurred in the past. It also has one drawback. Since the numbers coming out in each draw are random, there is no way to control what comes out in the next draw based on what was just drawn. Without the correlations, you could draw inflation at 5% and cash at 3%, which really does not occur in the economy.

Running the Simulation

Users can select to simulate returns and wealth values for assets and created portfolios. We begin with the expected return, standard deviation, and correlation of the assets. The returns for each asset are assumed to be lognormally distributed. The lognormal distribution is skewed to the right. That is, the expected value, or mean, is greater than the median. Furthermore, if returns are lognormally distributed, returns cannot fall below negative 100%. The expected return and a seed value are used to generate the first simulated return number for each asset class and inflation value (if selected). The seed value is used for randomness, as a starting point for random number generation, and is the same for every forecast that is simulated. You can specify the number of simulated runs for each asset each period, ranging between 500 and 5,000, generating expected returns for the desired number of periods. (Simulated returns for portfolios are computed by weighting the simulated asset returns accordingly.) All simulated returns take correlation into consideration, which prevents simulations of returns from being abnormally correlated. For example, you will not see a simulated 2% inflation value and a simulated 14% return on a government Treasury bill. The correlation matrix helps keep the consistency of the forecast.

If "Inflation-adjust returns and wealth values" is selected, the user must select an "inflation" asset to be simulated. Each period, the simulated inflation is subtracted from each asset's simulated returns to arrive at the "real simulated returns."

The simulated returns are sorted (highest to lowest) and organized into probability percentile ranges (Figures 2 and 3). The simulated returns are then used to calculate wealth values. We adjust the wealth values by the cash flows, then sort all wealth values into probability percentile ranges. Cash inflows are entered at the end of the period. Cash outflows are taken out at the beginning of the period.

The last step is to analyze and evaluate the output and to make any necessary adjustments to the inputs. This is an extremely important step that should not be overlooked.

Comparing Data – MVO vs. Monte Carlo Simulation

It is very important to analyze your data. Figures 2 and 3 show an example of possible differences that may occur when using a Monte Carlo simulation instead of MVO. Monte Carlo simulation includes a factor of uncertainty that MVO does not, which is why there are differences in the output.

The percentile tables are calculated using a lognormal distribution. You can see the range of compounded return and wealth possibilities for the portfolio over the entire investment horizon. The numbers displayed do not represent a guaranteed amount, but rather a range in which the return or wealth value for a particular year may fall.

For example, in year 1 there is a 5% chance of achieving a 55.10% return (MVO, Portfolio A) or higher. There is a 5% chance of having a -16.94% return or lower. And there is a 90% chance of earning between a 55.10% and -16.94% return (Figure 2).

The wealth percentiles table (Figure 3) can be read the same way, except instead of return values you are viewing wealth values. Portfolio A has an initial wealth value equal to \$100,000; does not include cash flows; and inflation was included.

Return Percentiles: Portfolio A

	1 year		5 year		10 year		25 year	
	MVO Portfolio A	Simulated Portfolio A	MVO Portfolio A	Simulated Portfolio A	MVO Portfolio A	Simulated Portfolio A	MVO Portfolio A	Simulated Portfolio A
95th Percentile	55.10	54.81	30.51	30.15	25.28	24.90	20.82	20.87
67th Percentile	23.39	23.46	17.82	17.83	16.54	16.54	15.41	15.54
50th Percentile	13.50	13.47	13.50	13.47	13.50	13.47	13.50	13.50
33rd Percentile	4.41	4.29	9.34	9.48	10.54	10.62	11.62	11.62
5th Percentile	-16.94	-16.50	-1.29	-0.96	2.83	3.13	6.63	6.56

Figure 2 Return Percentiles Table

Wealth Percentiles: Portfolio A

	1 year		5 year		10 year		25 year	
	MVO Portfolio A	Simulated Portfolio A	MVO Portfolio A	Simulated Portfolio A	MVO Portfolio A	Simulated Portfolio A	MVO Portfolio A	Simulated Portfolio A
95th Percentile	\$155,101	\$154,805	\$378,672	\$373,409	\$952,581	\$924,085	\$11,303,865	\$11,437,152
67th Percentile	\$123,389	\$123,462	\$227,061	\$227,159	\$462,142	\$461,980	\$3,602,051	\$3,699,849
50th Percentile	\$113,503	\$113,474	\$188,382	\$188,088	\$354,879	\$353,985	\$2,372,469	\$2,370,514
33rd Percentile	\$104,409	\$104,289	\$156,293	\$157,245	\$272,512	\$274,279	\$1,562,613	\$1,560,835
5th Percentile	\$83,062	\$83,503	\$93,717	\$95,303	\$132,208	\$136,077	\$497,937	\$489,591

Figure 3 Wealth Percentiles Table

Limitations of Monte Carlo Simulation

While Monte Carlo simulation has its fair share of benefits, as with other mathematical models, it also has its limitations. Simulations can lead to misleading results if inappropriate inputs are entered into the model. As was discussed earlier, the simulation process begins with entering asset class or portfolio returns, standard deviations, and correlations. When cash flows are added to the analysis they may be adjusted for inflation, which can present another possible problem if an unrealistic inflation rate is assumed. The user should be prepared to make the necessary adjustments if the results that are generated seem out of line.

While Monte Carlo does a fine job of illustrating the wide variance of possible results and the probability of success or failure over thousands of "different market environments," it may not consider the consequences based on the "applicable market environment" that exists over an investor's lifetime. There are a number of unknown factors that cannot truly be accounted for.

Monte Carlo simulation does not model serial correlations. This means the numbers coming out in each draw are random; there is no way to control what comes out in the next draw based on what was just drawn. For example, inflation could be 3% in one period and 10% in the next. Behavior like this just does not occur in the economy.

Summary

One thing to keep in mind is that projecting the future is no easy task. The future is unknowable. While Monte Carlo simulation can produce possible scenarios of the future and help investors make better decisions, it does not help investors make perfect decisions. There is an important point to remember when contemplating and choosing among the various simulation-based products. The quality of the forecast is directly related to the quality of the technique and the inputs used.

References

Ibbotson Product Support. Article # 374 Methods for generating simulations (Monte Carlo) <https://ibbotson.custhelp.com> (03 Nov. 2004).

Ibbotson Associates, "Wealth Forecasting with Monte Carlo Simulation." *Stocks, Bonds, Bills, and Inflation 2005 Yearbook*. Chicago, Ibbotson Associates, 2005.